PHYSICAL REVIEW E VOLUME 50, NUMBER 5 NOVEMBER 1994

Bifurcation of resonance islands and Landau damping in the double-rf system

J.Y. Liu,[1] M. Ball,[1] B. Brabson,[1] J. Budnick,[1] D.D. Caussyn,[1] G. East,[1] M. Ellison,[1] B. Hamilton,[1] W.P. Jones,[1] X. Kang,[1] S.Y. Lee,[1] D. Li,[1] K.Y. Ng,[2] A. Riabko,[1] D. Rich,[1] T. Sloan,[1] and L. Wang[1]
[1]*Indiana University Cyclotron Facility, Indiana University, Bloomington, Indiana 47405*
[2]*Fermilab, Box 500, Batavia, Illinois 60510*

(Received 27 June 1994; revised manuscript received 26 September 1994)

The attractors of a double-rf system subject to rf phase modulation, in the presence of a weak damping force, were measured as a function of the modulation frequency. We found that the phase amplitudes of the attractors followed a simple predictable path related to the synchrotron tune of the double-rf system. These attractors were found to bifurcate at a modulation frequency near the maximum synchrotron frequency. We also found that the coherent synchrotron oscillations decohered rapidly at small synchrotron amplitudes but showed little decoherence at large synchrotron amplitudes. The experimental result has some implications for the Landau damping of coherent beam instabilities.

PACS number(s): 29.27.Bd, 41.75.−i, 03.20.+i, 05.45.+b

(i) Introduction. The double-rf system has been used to alleviate the space charge effect by reducing the peak current [1,2]. It has also been used to overcome multibunch instabilities by modifying the time structure of the bunch, which changes the effective impedance experienced by the beam, and more importantly to increase the synchrotron tune spread of the beam for achieving an enhanced Landau damping [3,4]. Furthermore, a secondary high frequency rf system has also been used for controlled longitudinal beam emittance dilution [5].

In the past few years, there have been some theoretical studies on the double-rf system for small amplitude synchrotron oscillations [6], which are valid only for synchrotron phase amplitude $\hat{\phi}\leq 50°$. More recently, a method has been advanced to solve the double-rf system, without small amplitude approximations, in the presence of external coherent harmonic modulations [7]. In particular, an analytic solution has been obtained for the case where the harmonic ratio is 2 [8]. However, there are few experimental measurements of the properties of a double-rf system. To understand the particle motion in the double-rf system, it is important to measure characteristic properties of the dynamical system, and compare the experimental data with theory. This paper reports results of beam dynamics experiments done with a double-rf system.

(ii) Experimental technique and calibration. The experimental procedure started with 90-MeV H_2^+ ions strip-injected into the Indiana University Cyclotron Facility (IUCF) Cooler Ring, resulting in a proton kinetic energy of 45 MeV. The revolution frequency for the synchronous particle, f_0, was 1.031 68 MHz. The frequency of the primary rf cavity was 3.095 04 MHz, and for the secondary one, it was 6.190 08 MHz, with harmonic numbers $h_1=3$ and $h_2=6$, respectively. The ratio of the harmonic numbers was chosen to be equal to 2 so that the resulting double-rf system had the largest bucket area [7,8]. The primary rf voltage was set at about 92.7 V, which resulted in a synchrotron frequency of about 679 Hz while operating with the primary rf cavity alone. There were three bunches in the ring with a total current of about 200 μA, or equivalently 4×10^8 protons per bunch. The accelerator was operated with a cycle time of 10 s. The injected beam was electron-cooled for about 3 s. The cooling rate has been previously measured to be about 3 ± 1 s^{-1} [9,10], which is equivalent to a cooling time of about 300 ms.

Since our data acquisition system was capable of digitizing the six-dimensional (6D) phase space variables for only one bunch per revolution in the ring, a transverse dipole kicker was used to kick two bunches out of the aperture leaving a single bunch in the cooler ring. The full width at half maximum bunch length for the double-rf system was typically about 6.8 m (or 75 ns).

The Hamiltonian for the double-rf system with a harmonic ratio of 2 is given by

$$H_0=\tfrac{1}{2}\nu_s\delta^2+\nu_s[(1-\cos\phi)-(r/2)(1-\cos 2\phi)], \quad (1)$$

where $\nu_s = (h_1 eV_1|\eta|/2\pi\beta^2 E)^{1/2}$ is the small amplitude synchrotron tune of the primary rf system alone, $r=V_2/V_1$ is the ratio of the rf voltages. The synchrotron tune is the number of synchrotron oscillations per revolution. The conjugate phase space coordinates (ϕ,δ) are respectively the phase of the particle relative to that of the synchronous particle and the normalized off-momentum variable $\delta=-(h|\eta|/\nu_s)\times(\Delta p/p_0)$, where $\eta=-0.86$ is the phase slip factor.

The rf voltages of the two cavities were calibrated individually by measuring their synchrotron frequencies, when each cavity was operated alone. One should, however, keep in mind that the measured synchrotron tune depends on the amplitude of synchrotron oscillations. Additionally, when the two cavities were operating together, the voltage ratio could be calibrated by measuring the shape of the potential well. With electron cooling, the protons were dampened into the bottom of the potential well because the equilibrium beam profile followed a Hamiltonian contour. In particular, the potential well has two dips when $r>0.5$. This was the case for the particles trapped in the potential well shown in Fig. 1, where traces of sum signals from a beam position monitor (BPM) were accumulated with an oscilloscope triggered at the revolution frequency. The rf wave forms from pickup loops for both rf cavities are also shown. Note that the sum signal from a BPM is proportional to the beam charge. This

FIG. 1. The trace of a BPM sum signal accumulated on an oscilloscope and triggered at the revolution frequency. Note here that there was only one bunch present after a horizontal kicker was used to kick two bunches away. The rf wave forms for the two cavities are also shown. The beamlet phase separation was used to calibrate the voltage ratio.

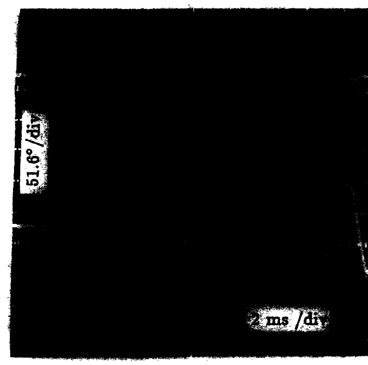

FIG. 2. The signal from a phase detector shown as a function of time for 40° and 90° phase kicks. The rf voltage ratio was about 0.3 and the synchrotron frequency of the primary rf system was about 679 Hz.

beam profile has two peaks separated by about 60.4 ns, which is the result for the voltage ratio of $r \approx 0.60$. The relative intensity of these two peaks depends on the relative phases of the two rf cavities. Using this sensitivity, the relative phase of two cavities can be calibrated and adjusted. At the same time, by measuring the separation of the two peaks inside a rf bucket the ratio of the two rf voltages could also be calibrated. These different calibration methods agreed to within 5%.

To measure the synchrotron tune of the double-rf system, the bunched beam was kicked longitudinally by phase shifting the control signal for both rf cavities, and the resulting phase oscillations were digitized. The principle of the phase shifters of our rf systems has been discussed before [9]. The control voltage for the phase shifters vs the actual phase shift was experimentally calibrated. Both the phase error due to control nonlinearity and the parasitic amplitude modulation of the IUCF Cooler rf systems have been kept to less than 10%. The phase lock feedback loop, which normally locks the rf cavity to the beam, was switched off for our experiment. The response time of the step phase shifts was primarily limited by the inertia of the rf cavities which both have a quality factor Q of about 40. The phase shift was accomplished by applying a square wave with frequency 0.2 Hz, resulting in a rf phase shift with a 2.5 s duration. The magnitude of the phase shift was varied by adjusting the amplitude of the square wave.

It is important that the phase shifts in both cavities are in a proper ratio so that the shape of the potential well remains unchanged. For this reason, a voltage divider was used to divide the applied voltage between the two cavities. Once the beam is phase kicked, the subsequent beam-centroid displacements $\Delta x_{\rm co}$ were measured with the Δ signal of a BPM, located in a region of high dispersion $D_x \approx 3.9$ m to obtain the off-momentum variable given by $\Delta p/p = \Delta x_{\rm co}/D_x$. The signal from the BPM was passed through a 3 kHz low pass filter before digitization to remove effects due to coherent betatron oscillations and high frequency noise. The BPM had a rms position resolution of about 0.1 mm. Similarly, the sum signal from a BPM, passing through a 1.4 MHz low pass filter, was used to obtain the phase coordinate by comparing the signal with the rf wave form of a pickup loop in the primary rf cavity. Our present phase detector has a range of 720° with a resolution of about 0.2°. The 6D phase space coordinates were digitized in 10-revolution intervals for 16 384 points (or 163 840 revolutions). A Poincaré map can be obtained from the digitized data and the synchrotron tune can be obtained from a fast Fourier transform of the phase or the off-momentum variables.

For online diagnosis, the phase oscillations were displayed on an oscilloscope. Figure 2 shows examples of 40° and 90° kicks for a voltage ratio of $r \approx 0.3$, where the scope was triggered 4 ms before a phase kick was applied to the beam. Note that the synchrotron oscillations resulting from the 40° kick decohere rapidly, while those resulting from the 90° kick show little decoherence. Since the effective damping time of the electron cooling was of the order of 300 ms, the damping of the phase oscillations observed in the upper plot of Fig. 2 was not due to electron cooling but resulted from decoherence of a beam with a large tune spread. This rapid decoherence, resulting from a large tune spread within the bunch, may be important for the Landau damping of coherent instabilities.

The synchrotron tune of the Hamiltonian of Eq. (1) has been obtained analytically as a function of the maximum phase amplitude [8]. The solid lines in Fig. 3 show the synchrotron tunes for $r = 0.0$, 0.1, 0.2, 0.3, 0.4, and 0.5, respectively. The measured synchrotron tunes for $r \approx 0.3$ are shown as square symbols. Note that when the voltage ratio is 0.5, the synchrotron tune goes to zero as the synchrotron amplitude goes to zero. Since the effective tune spread of the beam is given by $\Delta Q_s = (dQ_s/dJ)\Delta J$, where ΔJ is the maximum width in action of the beam bunch, the synchrotron tune spread will be larger at the phase space locations with a larger dQ_s/dJ. Therefore the synchrotron tune spread of the beam is much larger for small amplitude oscillations than that for large amplitude oscillations. At $r = 0.5$, our experimental data indicated that small amplitude synchrotron oscillations decohered within one synchrotron period. Thus the double-rf system, with the proper voltage ratio, can provide large synchrotron tune spread for Landau damping. However, we note that the derivative of the synchrotron tune becomes zero at the phase amplitude of about 117°. This means that the beam could become unstable if affected by time dependent perturbations at the top of the synchrotron frequency.

(iii) Observation of parametric resonances and bifurca-

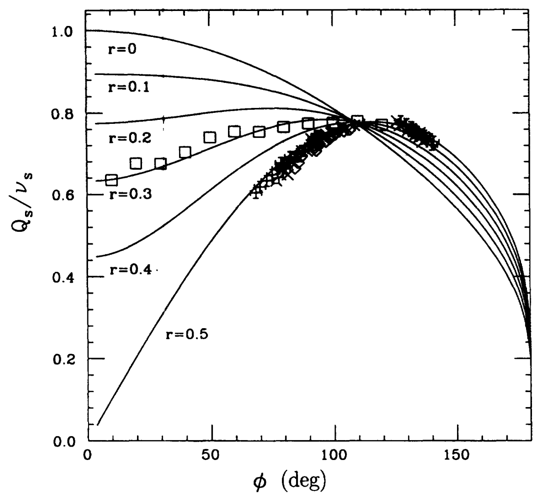

FIG. 3. The synchrotron tunes, normalized to the zero amplitude synchrotron tune of the primary rf system, are shown as a function of the maximum phase amplitude for the voltage ratios of $r=0.0, 0.1, 0.2, 0.3, 0.4$, and 0.5, respectively. The square symbols show the measured synchrotron tune for $r\approx 0.3$. Other symbols are the measured phase amplitudes of attractors when phase modulation was applied to the double-rf system at $r=0.5$ with phase modulation amplitudes varied from 2.5° to 5°.

tion. To study the particle beam stability, experimental measurements of beam response to time dependent perturbations [7–10] were performed. Adding a small sinusoidal wave to the square wave, which was used for phase shifting the rf cavities, the rf cavities were simultaneously phase shifted and sinusoidally modulated. In high energy synchrotrons, phase modulation may arise from synchrobetatron coupling, rf noise, or a wake field resulting from the longitudinal impedances. With phase modulation, the Hamiltonian becomes

$$H=H_0+\nu_m a\,\delta\cos\nu_m\theta, \tag{2}$$

where a is the modulation amplitude, ν_m is the modulation tune, which is the number of modulation oscillations per revolution.

In the limit of small perturbation with $a\ll 1$, the perturbation term in Eq. (2) can be expanded in action-angle variables of the unperturbed Hamiltonian H_0, where the action is $J=(1/2\pi)\oint\delta d\phi$. Therefore the "energy" of the unperturbed Hamiltonian, $H_0=E_0$, can be expressed as a function of the action J. The synchrotron tune is given by

$$Q_s(J)=dE_0/dJ. \tag{3}$$

Using the generating function $F_2(\phi,J)=\int_{\hat{\phi}}^{\phi}\delta(\phi')d\phi'$, where $\hat{\phi}$ is an extremum of the phase angle for a given torus, the angle variable is then given by $\psi = \partial F_2/\partial J = (\partial E_0/\partial J)\int_{\hat{\phi}}^{\phi}(\partial\delta/\partial E_0)\,d\phi'$.

We expand the perturbation in action-angle variables of the unperturbed Hamiltonian with

$$\delta=\sum_n g_n(J)e^{in\psi}, \tag{4}$$

where the strength function $g_n(J)$ can be obtained from the inverse Fourier transform. Using the symmetry of the unperturbed Hamiltonian, it is easy to prove that the phase modulation produces odd order resonances, while voltage modulation results in even order resonances [7,8]. We also note that the strength function obeys a sum rule [8]

$$J=[\nu_s/Q_s(J)]\sum_{n=-\infty}^{\infty}|g_n(J)|^2, \tag{5}$$

which is generally valid for Hamiltonian systems in which the kinetic energy depends quadratically on the momentum.

Inserting Eq. (4) into Eq. (2), one finds that the perturbation creates parametric resonance islands inside the rf bucket. The actual resonance strength for the nth harmonic resonance is given by $\nu_m a|g_n|$. When the perturbation is small, the condition for the nth order resonance islands is given by

$$\nu_m\approx nQ_s(J_r), \tag{6}$$

where J_r is the resonance action. In a weakly dissipative system such as the IUCF Cooler Ring, these stable fixed points become attractors. By measuring the phase amplitude of the attractor vs the modulation tune ν_m, one can effectively trace the $Q_s(J)$ across the rf bucket.

If Q_s has a maximum value $\hat{Q}_s$, then the solution of Eq. (6) will exhibit the characteristics of resonance bifurcation, i.e., resonance islands disappear when $\nu_m>\hat{Q}_s$. Since the island width is given by $\Delta J_r = 4(\nu_m a|g_n|/Q_s'|_{J=J_r})^{1/2}$, the resonance island size becomes large when $Q_s' = dQ_s/dJ|_{J=J_r}$ is small. Since the synchrotron tune of Eq. (1) for $r=0.5$ peaks at $\hat{Q}_s\approx 0.7786\nu_s$ at the maximum phase amplitude of 117°, and vanishes at both large and small phase amplitudes, the resonance condition of Eq. (6) for a given order n will occur at two different actions until the modulation tune reaches the peak synchrotron tune $\hat{Q}_s$. In a particular region of the modulation tune, one can observe two merging attractors. When the ν_m becomes larger than $n\hat{Q}_s$, there is no action which will satisfy Eq. (6), and all islands of order n suddenly disappear.

In our experiments, the synchrotron frequency of the primary rf system was 679 Hz, the phase modulation amplitude was varied from 2.5° to about 10°, and the phase modulation frequency was varied from about 400 Hz to about 1600 Hz. Figure 4(a) shows an example of the accumulated BPM sum signal traces on an oscilloscope, where the outer beamlet is 77.2 ns or 86.0° away from the central beamlet. The double peaks around the central peak were due to the synchrotron oscillations of particles trapped in the outer island, which was created by the phase modulation at 460 Hz. Using a fast sampling scope to obtain a single pass beam distribution, we observed two beamlets, shown in Fig. 4(b), for the modulation frequency of 495 Hz.

When the modulation frequency was increased beyond 500 Hz, two outer attractors along with a central flat potential well began to develop as shown in Fig. 5, where the modulation frequency was 505 Hz, and the measured phase coordinates of the attractors were 100° and 136°, respectively. When the modulation frequency was increased, these two attractors moved toward each other. The inner attractor

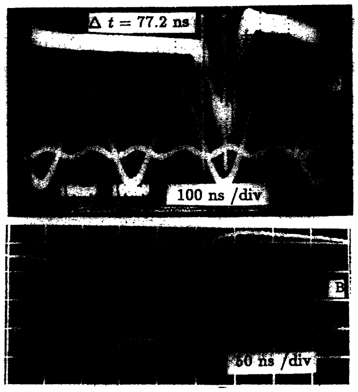

FIG. 4. (a) The bunch profile obtained from the accumulated BPM sum signals on an oscilloscope. The modulation frequency was 460 Hz, and there were only two beamlets. The observed two outer peaks were due to the synchrotron oscillations of particles trapped in the outer attractor. (b) The instantaneous longitudinal beam profile in a bucket obtained using a fast sampling oscilloscope. Note that there were only two beamlets. The modulation frequency was 495 Hz.

FIG. 5. The double peaks on both sides of the center attractor showed the existence of two approaching attractors at the modulation frequency of 505 Hz.

disappeared at about 512 Hz and the outer parametric island remained intact until about 530 Hz, depending on the modulation amplitude. The results of our many experimental runs are summarized in Fig. 3, where the modulation tune is plotted vs the amplitudes of the attractors observed on the oscilloscope. We found that the phase amplitudes of attractors followed the synchrotron tune of the unperturbed Hamiltonian with a weak dependence on the phase modulation amplitude. Similar behavior was observed for the third harmonic attractors, where resonance islands (attractors) were found to trace out $3Q_s(J)$.

(iv) Conclusion. In conclusion, we have measured two bifurcation branches for the attractors of a weakly dissipative double-rf system. We found that the attractors followed the characteristic tune of the unperturbed Hamiltonian. Around the $n=1$ synchrotron harmonic (dipole mode), the center region was stable provided that the phase modulation amplitude was less than 5°. This stability may result from the phase damping due to electron cooling. The confined stochasticity, observed in numerical simulations [7], seems not to affect the beam stability for the phase modulation amplitude less than 5°. For a phase modulation amplitude beyond 5°, the beam bunch becomes highly unstable even with the electron cooling. We also observed $n=3$ resonance (sextupole mode), which obeyed the condition of Eq. (6). The $n=2$ resonance was not observed for modulation amplitudes up to about 5°. In the double-rf system, small amplitude coherent oscillations were also observed to decohere rapidly. On the other hand, the coherent synchrotron oscillations would last for a long time if the kicked amplitude is larger than 90°. These results provide a criterion for Landau damping of small bunched beams in the double-rf system.

We thank X. Pei for his help in these experiments. Work supported in part by grants from the NSF PHY-9221402 and the DOE DE-FG02-93ER40801.

[1] G. Gelato *et al.*, in *Proceedings of the IEEE Particle Accelerator Conference, Washington, 1987* (IEEE, New York, 1987), p. 1298; J.M. Baillod *et al.*, IEEE Trans. Nucl. Sci. **NS-30**, 3499 (1983).

[2] In one of our beam dynamics experiments at the IUCF Cooler Ring, we have observed a fourfold increase in beam intensity when the double-rf system is used in conjunction with optimized electron cooling.

[3] R. Averill *et al.*, in *Proceedings of the 8th Conference on High Energy Accelerators*, edited by M.H. Blewett (CERN, Geneva, 1971), p. 301.

[4] P. Bramham *et al.*, IEEE Trans. Nucl. Sci. **NS-24**, 1490 (1977).

[5] V.V. Balandin *et al.*, Part. Accel. **35**, 1 (1991); R. Cappi *et al.*, CERN Report No. CERN/PS 92-40 (RF), 1992 (unpublished).

[6] A. Hofmann and S. Myers, in *Proceedings of the 11th Conference on High Energy Accelerators* (Birkhauser Verlag, Boston, 1980), p. 610; S. Krinsky and J.M. Wang, Part. Accel. **17**, 109 (1984); J. Wei, in *Proceedings of the Third European Particle Accelerator Conference* (Editions Frontiéres, France, 1992); A. Pauluhn, Deutsches Electron Synchrotron Report No. HERA 93-02, 1993 (unpublished).

[7] S.Y. Lee *et al.*, Phys. Rev. E **49**, 5717 (1994).

[8] J.Y. Liu *et al.* (unpublished).

[9] M. Ellison *et al.*, Phys. Rev. Lett. **70**, 591 (1993); H. Huang *et al.*, Phys. Rev. E **48**, 4678 (1993).

[10] Y. Wang *et al.*, Phys. Rev. E **49**, 1610 (1994); M. Syphers *et al.*, Phys. Rev. Lett. **71**, 719 (1993); D. Li *et al.*, Phys. Rev. E **48**, R1638 (1993).

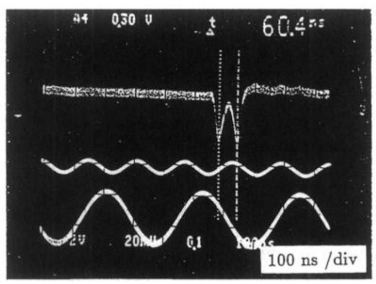

FIG. 1. The trace of a BPM sum signal accumulated on an oscilloscope and triggered at the revolution frequency. Note here that there was only one bunch present after a horizontal kicker was used to kick two bunches away. The rf wave forms for the two cavities are also shown. The beamlet phase separation was used to calibrate the voltage ratio.

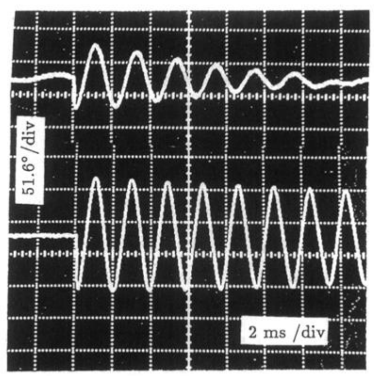

FIG. 2. The signal from a phase detector shown as a function of time for 40° and 90° phase kicks. The rf voltage ratio was about 0.3 and the synchrotron frequency of the primary rf system was about 679 Hz.

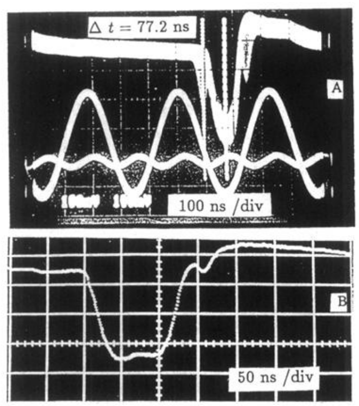

FIG. 4. (a) The bunch profile obtained from the accumulated BPM sum signals on an oscilloscope. The modulation frequency was 460 Hz, and there were only two beamlets. The observed two outer peaks were due to the synchrotron oscillations of particles trapped in the outer attractor. (b) The instantaneous longitudinal beam profile in a bucket obtained using a fast sampling oscilloscope. Note that there were only two beamlets. The modulation frequency was 495 Hz.

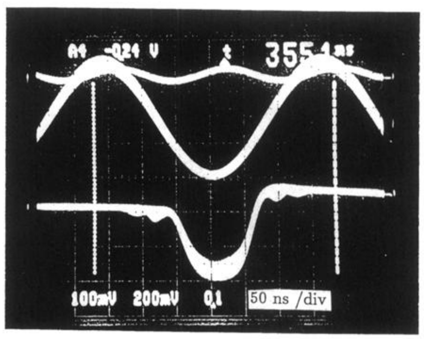

FIG. 5. The double peaks on both sides of the center attractor showed the existence of two approaching attractors at the modulation frequency of 505 Hz.